Work Done by a variable force

Fact (Recall) —

$$\text{Work Done} = \underbrace{\text{Force}}_{\text{component of force in the direction of travel}} \times \text{Distance}$$

$$\sum \text{Work Done} = \sum \underbrace{\text{Force}}_{\text{component of force in the direction of travel}} \times \underbrace{\delta x}_{\text{distance}}$$

$$\text{Work Done} = \int_{x_1}^{x_2} \text{Force}\,\mathrm{d}x$$

$$\text{Work Done} = \int_{\mathbf{x}_1}^{\mathbf{x}_2} \mathbf{F}\cdot\mathrm{d}\mathbf{x}$$

Example

A car of mass 1020 kg moves from rest at A on a horizontal surface. The driving force is constant at 1800 N and resistance to motion is modelled as $\frac{x^2}{8}$N. The car moves 120 m to B.

(a) Find the work done by the driving force and the work done against resistance as the car travels from A to B.

(b) Find the speed of the car at B

Example

An object is moving in a horizontal straight line against a resistive force that is directly proportional to its distance from its starting point, $f(x) = kx$. If the work done against resistance as the object travels from the origin $x = 0$ m, to a point 15 m, is 337.5 kJ.

(a) find the magnitude of k

(b) state the units of k.

Hooke's Law

Fact (Hooke's Law) — Hooke's law states that

$$\text{Force} \propto \underbrace{\text{compression}}_{\text{spring}} \; / \; \underbrace{\text{extension}}_{\text{string}}$$

$$\text{Force} = \frac{\lambda \quad \times \quad x}{l}$$

What are the units of λ?

Example

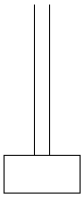
A particle, of mass 5 kg, is suspended from a spring, of natural length 0.2 m and modulus of elasticity 40 N. Find the extension of the spring when the particle is in equilibrium

Example

A light elastic spring, whcih has modulus of elasticity 85 N and natural length 1.8 m, has one end attached at a fixed point A. A horizontal force, of magnitude 40 N, is applied to the spring causing a compression. The spring rests in equilibrium, find the distance the spring is compressed from it's natural length.

Example

An object P of mass 10 kg is attached to the lower ends of two light elastic strings. One string is of natural length 0.5 m with modulus of elasticity 25 N. The other string is of natural length 0.4 m with modulus of elasticity 30 N. The free ends of the strings are attached to a point A and P hands vertically below A. Find the distance AP.

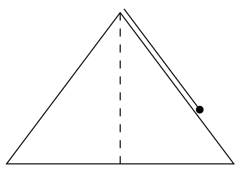


Example

A right circular cone C of height 4 m and base radius 3 m has its base fixed to a horizontal plane. One end of a light elastic string of natural length 2 m and modulus of elasticity 32 N is fixed to the vertex of C. The other end of the string is attached to a particle P of mass 2.5 kg. P moves in a horizontal circle with constant speed and in contact with the smooth curved surface of C. The extension of the string is 1.5 m.

(a) Find the tension in the string. [2]

(b) Find the speed of P. [7]



Conservation of Energy

Example

Find the work done when a light elastic string of natural length 1.2 m and modulus of elasticity 80 N is stretched from a length of 1.5 m to 1.8 m.

Fact — For an elastic object, being stretched (compressed) from x_1 to x_2 the change in elastic potential energy is $\frac{\lambda}{2l}\left(x_2^2 - x_1^2\right)$. Measuring from the natural length, we can say the EPE is $\frac{\lambda x^2}{2l}$

Example

An elastic string has natural length a. One end is fixed. A particle of mass $2m$ is attached to the free end hangs in equilibrium, with the length of the string $3a$. Find the elastic potential energy stored in the string.

Example

One end of a light elastic string of natural length 0.8 m and modulus of elasticity 50 N is attached to a fixed point O. A particle P of mass 1.5 kg is attached to the other side of the string. P is released rom rest at O and falls vertically. Assuming there is no air resistance find:

(a) the extension of the string when P is at its lowest position

(b) the acceleration of P at it's lowest position

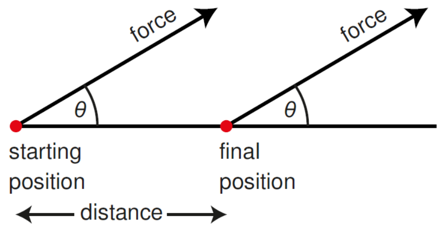
Example

A light elastic spring, of natural length 1 m and modulus of elasticity 20 N, has one end attached to a fixed point A. A particle of mass 2 kg is attached to the other end of the spring and is held at a point B which is 0.8 m vertically below A. The particle is projected vertically downwards from B with speed $2\,\text{m}\,\text{s}^{-1}$.

Find the distance it falls before first coming to rest.

Work Done, Energy and Power in two dimensions

If a constant force is directed at an angle θ to the direction of motion then:



$$\text{work done} = \text{force} \times \cos\theta \times \text{distance}$$

Fact — The work done by a (constant) force $\mathbf{F}$ which causes a displacement $\mathbf{x}$ is:

work done =

Example

A force $(2\mathbf{i} + \mathbf{j})$N is acting on an object that moves from A, with position vector $(3\mathbf{i} + \mathbf{j})$m, to B with position vector $(4\mathbf{i} + 2\mathbf{j})$m. Find the work done by the force.

Example

An object, of mass 2kg, is at rest at A when a constant force $(2\mathbf{i} - \mathbf{j})$N causes the object to move to B. A has position vector $(\mathbf{i} - 2\mathbf{j})$m and B has position vector $(2\mathbf{i} - 6\mathbf{j})$m. Given that no other forces act on the object, use the work-energy principle to find the speed of the object at B.

Definition.

- Work done on an object by a *propulsive* force is __________ in sign.
- Work done on an object by a *resistive* force is __________ in sign.

Example

A force $(-3\mathbf{i}+2\mathbf{j})$N is acting on an object that moves from A, with position vector $(\mathbf{i}-\mathbf{j})$m, to B with position vector $(5\mathbf{i}+4\mathbf{j})$m. Find the work done by the force and state whether the force is propulsive or resistive.

Fact — The kinetic energy of an object of mass m kg moving with velocity $\mathbf{v}$ ms^{-1} is defined as

kinetic energy =

where $\mathbf{v}\cdot\mathbf{v}$ is the scalar product of velocity with itself.

Example

A rocket of mass 2 tonnes is moving with velocity $(2\mathbf{i}+5\mathbf{j})$ms^{-1}. Find the kinetic energy of the rocket, giving your answer in kJ.

Example

A small object of mass 500g accelerates across a horizontal surface due to the action of a force that is acting at an angle to the resulting displacement. The driving force is $(5\mathbf{i}+3\mathbf{j})$N, and the displacement is $(65\mathbf{i}+80\mathbf{j})$m. Given that the starting speed is 4.5ms^{-1}, find the final speed of the object.

Fact — The power of an engine producing a driving force $\mathbf{F}$ N on an object moving with velocity $\mathbf{v}$ ms^{-1} can be calculated from the formula:

$$\text{power} =$$

Example

A vehicle is moving under the action of a driving force $(35\mathbf{i} - 60\mathbf{j})$ N in a horizontal plane with velocity $(5\mathbf{i} - 7\mathbf{j})\text{ms}^{-1}$. Given that no other forces are acting on the vehicle, find the power of the vehicle engine.